METHOD OF SUBSTITION

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Integrand: The function to which the integration is applied the integrand. and the function obtained as a result of integration is called integral.

Q) Evaluate $\int x^2 (1+x^3)^{3/2} dx$

Solution: Given, $x^2 (1+x^3)^{3/2} dx$

Let $(1+x^3) = z$

$0 + 3x^2 = \frac{dz}{dx}$

$\therefore dx = \frac{dz}{3x^2}$

$= \cancel{x^2} \cdot z^{3/2} \cdot \frac{dz}{3\cancel{x^2}}$

$= \frac{z^{3/2}}{3} \cdot dz$

$= \frac{1}{3} \cdot \frac{z^{5/2}}{5/2} = \frac{2}{15} z^{5/2}$

$= \frac{2}{15} (1+x^3)^{5/2} + c$ Ans

Q) Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

Solution, Given,

$\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

$\therefore \frac{\sin t}{t} \cdot \frac{3}{2} \frac{dt}{x^{3/2}}$

Let $\sqrt{x} = t$

$\frac{x^{3/2}}{3/2} \therefore \frac{dt}{dx}$

$\therefore dx = \frac{\frac{3}{2} dt}{x^{3/2}} = \frac{3}{2x^{3/2}} \cdot dt$

$\therefore \sqrt{x} = z$

$\frac{1}{2\sqrt{x}} = \frac{dz}{dx}$

$\therefore dx = 2\sqrt{x}\,dz$

$\therefore \int \frac{\sin z}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\,dz$

$= \int 2 \sin z\,dz$

On integrating we get

$= -2\cos z + C$

$= -2\cos\sqrt{x} + C$ Ans

Q $\int \frac{dx}{x(a+b\log x)}$

Let $a+b\log x = z$

$0 + b\cdot\frac{1}{x} = \frac{dz}{dx}$

$\therefore dx = \frac{x}{b}\,dz$

On putting the value of dx we get

$\int \frac{1}{\cancel{x}\cdot z} \cdot \frac{\cancel{x}}{b}\,dz$

$= \int \frac{1}{b\,z}\,dz$

$= \frac{1}{b}\int\frac{1}{z}\,dz = \frac{1}{b}\log z$

$= \frac{1}{b}\log(a+b\log x) + C$ Ans

Q Evaluate $\int \frac{dx}{x^2\sqrt{1+x^2}}$

Solution. Given, $\int \frac{dx}{x^2\sqrt{1+x^2}}$

Let $x = \tan\theta$

$1 = \sec^2\theta \frac{d\theta}{dx}$

$\therefore \int \frac{\sec^2\theta \, d\theta}{\tan^2\theta \sqrt{1+\tan^2\theta}}$

$= \frac{\sec^2\theta}{\tan^2\theta \cdot \sec\theta} \, d\theta$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= \frac{\sec\theta}{\tan^2\theta} d\theta$

$= \frac{1}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} \, d\theta$

$= \frac{\cos\theta}{\sin^2\theta} \, d\theta$ ~~Let $\sin\theta = t$, $\cos\theta = \frac{dt}{d\theta}$~~

~~Let $\sin\theta = t$~~

~~$\cos\theta = \frac{dt}{d\theta}$~~

~~$\therefore d\theta = \frac{dt}{\cos\theta}$~~

~~$= \frac{\cos\theta}{t^2} \cdot \frac{dt}{\cos\theta}$~~

~~$= \int \frac{dt}{t^2}$~~

~~$= \frac{t^{-2+1}}{-2+1}$~~

~~$= \frac{t^{-1}}{-1} = -t^{-1}$~~

~~$= -\frac{1}{t} =$~~

~~$= \frac{-1}{\sin\theta} = -\operatorname{cosec}\theta = -\operatorname{cosec} si$~~

Let $\sin\theta = t$

$\cos\theta = \frac{dt}{d\theta}$

$\therefore d\theta = \frac{dt}{\cos\theta}$

$= \frac{\cos\theta}{t^2} \cdot \frac{dt}{\cos\theta}$

$= -\frac{1}{t} dt = \frac{1}{t^2} \cdot dt$

$= -\frac{1}{}$ on integrating we get –

$= \frac{-1}{t} = \frac{-1}{\sin\theta}$

$= -\operatorname{cosec}\theta$

$\therefore$ $-\operatorname{cosec}\theta$ [$\therefore$ $\tan\theta = x$

$= -\frac{\sqrt{1+x^2}}{x} + C$ Ans $\therefore \sin\theta = \frac{x}{\sqrt{1+x^2}}$

Q) Evaluate $\int \sin^5 x\, dx$

Solution. Given,

$\int \sin^5 x \cdot dx$

$= \sin^4 x \cdot \sin x\, dx$

$= (\sin^2 x)^2 \cdot \sin x \cdot dx$

$= (1-\cos^2 x)^2 \sin x \cdot dx$

put $\cos x = t$

$-\sin x = \frac{dt}{dx}$

$\therefore dx = \frac{-dt}{\sin x}$

$= -(1-t^2)^2 \cdot \cancel{\sin x} \cdot \frac{dt}{\cancel{\sin x}}$

$= -(1-t^2)^2\, dt$

$= -[1-2t^2+t^4]\, dt$

$= 2t^2 - 1 - t^4$

On integrating we get,

$= \frac{2t^3}{3} - t - \frac{t^5}{5}$

$= \frac{2\cdot\cos^3 x}{3} - \cos x - \frac{\cos^5 x}{5}$ Ans

Q) Evaluate, $\int \tan^4 x\, dx$

Solution: $\tan^4 x$

$= \tan^2 x \cdot \tan^2 x$

$= \tan^2 x\, (\sec^2 x - 1)$ [$\therefore$ $\sec^2 x - \tan^2 x$